<u>a-g Geometry (Ca Standards)</u>

<u>Note to ES and EE:</u>
Students must use one of the approved textbooks listed below to meet the requirements of this course. Other textbooks can be used to supplement the course. Contact the Math Department Chair for assistance with using supplemental material.

Course Description

Overview:

Geometry is the second course in the high school math series. The course builds on the students' experiences from the middle school grades and includes standards from the conceptual categories of Geometry and Statistics and Probability. Problem solving and application of geometry concepts will require the use of algebra concepts that students have learned in Algebra 1 or Integrated Math 1. Students will learn to apply inductive and deductive reasoning skills and formalize the development of proof in an extensive study of triangle congruence and similarity, as well as proofs of theorems related to circles. They will develop and apply formulas for circumference, area, and volume. They will also apply the Pythagorean Theorem to the coordinate plane and extend their work with probability that was begun in the middle grades.

Prerequisites: Algebra 1 or Integrated Math 1 - Required
Corequisites: None

Reasoning and Proof

The unit begins with the foundations of Euclidean geometry, understanding the essential postulates regarding points, lines and planes, and the basic tools of construction--the compass and straightedge. Building on this foundation, students are introduced to angles and relationships among pairs of angles. Students are then introduced to inductive and deductive reasoning. They examine patterns in number sequences and in sequences of geometric figures. They learn to make a conjecture and prove it false by finding a counterexample or prove it to be true using deductive reasoning in a mathematical proof. Students will begin proofs with algebraic equations and then prove geometric relationships using given information, definitions, properties, postulates, and theorems. Students will learn the logic of conditional statements and how they are expressed in definitions, properties, postulates and theorems. The unit then expands on the students' knowledge of lines and angles and proof writing. Students explore the idea that not all lines and not all planes intersect. When a line intersects two or more lines, the angles formed at the intersection points create special angle pairs. The special angle pairs formed by parallel lines and a transversal are either congruent or supplementary. Students will prove and use theorems about certain pairs of angles that can be used to decide whether two lines are parallel or perpendicular.

Sample Assignment:

Students will produce a portfolio of basic compass and straightedge constructions: a segment congruent to a given segment, an angles congruent to a given angle, perpendicular bisector of a segment, an angle bisector, and parallel lines. They will check the accuracy of their constructions using protractors and rulers, and answer the following questions:

- What are the two tools used to make constructions?
- How can a compass be used as a measuring tool?
- Describe the difference in accuracy between sketching a figure, drawing a figure with a ruler and protractor, and constructing a figure.

Students learn the meaning of bisecting both a segment and an angle. They learn the meaning of perpendicular, and they become familiar with the tools of geometry and the limitations of the accuracy of each tool.

Triangle Congruence and Properties of Triangles

This unit builds upon the students' understanding and skills related to angles and triangles. Students will first learn that two figures are congruent if all of the corresponding parts are congruent. Students will then learn that two triangles can be proven congruent without proving all of the corresponding parts congruent. Specifically, they are congruent if three pairs of corresponding sides (SSS) are congruent, two pairs of corresponding sides and one pair of corresponding angles (SAS or HL) are congruent, or one pair of corresponding sides and two pairs of corresponding angles (ASA or AAS) are congruent. Students combine visual and deductive skills to determine triangle congruence, determining which postulate or theorem to use to prove triangle congruence, and learn to analyze diagrams that may involve overlapping triangles. Students build their deductive arguments to prove consequences of congruent triangles, such as two congruent triangles having congruent third sides or altitudes. Expanding on their knowledge of triangle congruency, students will use triangle congruency to prove the Perpendicular Bisector Theorem and the Angle Bisector Theorem. They will make conjectures based on an inquiry lesson regarding the midsegments of triangles and then learn to use coordinate geometry to prove this conjecture, known as the Triangle Midsegment Theorem. They will learn about the special points of concurrency of a triangle the three perpendicular bisectors indicate the circumcenter, the angle bisectors indicate the incenter, and the altitudes indicate the orthocenter. Students will study the

Pythagorean Theorem and use it to determine if a triangle is right, acute or obtuse. They will then use the Pythagorean Theorem to derive the ratios of the side lengths of the special right triangles, 30°- 60°- 90° and $45^{\circ}-$¿ $45^{\circ}-$¿ 90° . They will use the ratios to find a missing side length or to solve problems involving these special triangles.

Sample Assignment: "Pieces of Proof"

Students work in groups to arrange the statements and reasons of a two-column proof related to triangle congruence. Each group is given an envelope with the diagram, Given and Prove statements, as well as strips of paper that are either statements or reasons or the proof. Students are given the following directions:

1. Determine which strips are statements, and which are reasons
2. Determine which statements are "given" to start constructing the proof
3. Work together in a group to determine a logical argument for the proof. Each student then copies the complete proof, including the diagram.

In this activity, students identify the properties that exist in a given figure and apply postulates and theorems to build a formal proof involving triangle congruency. They work in a group and practice forming a viable argument while critiquing the reasoning of others.

Polygons and Quadrilaterals

In this unit, students examine the properties of quadrilaterals. They use the Triangle Angle Sum Theorem learned in the previous unit to derive a formula for finding the sum of the measures of the interior angles of any polygon, based on the number of sides. Students will use the properties of parallel and perpendicular lines and diagonals to classify quadrilaterals. They will learn that parallelograms have special properties regarding their sides, angles and diagonals. They will also learn that if a quadrilaterals sides, angles, and diagonals have certain properties, then the quadrilateral is a parallelogram. Students will also learn about the special properties of rhombuses, squares, rectangles and trapezoids. For figures placed on the coordinate plane, students will use the formulas for slope, distance and midpoint to classify figures and prove geometric relationships. Students will learn to assign variables to name coordinates of a figure on the coordinate plane, allowing them to prove relationships for a general case.

Sample Assignment:

Students are given the coordinates of three points on the coordinate plane. They use the definition and properties of parallelograms as well as the distance formula and slope to determine all possible coordinates of the fourth vertex.

This activity will reinforce the concept that slope can be used on the coordinate plane to determine if two segments are parallel. Students apply the slope formula and the definition of a parallelogram to find the missing points in this assignment.

Similarity and Right Triangle Trigonometry

This unit expands on students' understanding of similar figures figures whose corresponding side lengths are proportional and whose corresponding angles are congruent. Students use ratios and proportions to determine whether two polygons are similar and to find unknown side lengths of similar figures. They will learn to prove triangles similar based on the relationship of two or three pairs of corresponding parts: Angle-Angle, Side-Angle-Side, or Side-Side-Side. Students will apply this knowledge to problems involving indirect measurement, after first proving two triangles similar in a realworld diagram. Students will also learn that the altitude drawn to the hypotenuse of a right triangle forms three similar right triangles. The relationship between the segments in right triangles can be used for indirect measurement as well. Students explore the properties of similarity transformations, dilations, which either enlarge or reduce a figure according to a scale factor. Also, compositions of rigid motions and dilations can be used to understand the properties of similarity. Two figures are similar if there is a similarity transformation that maps one to the other. Students will then extend their understanding of similar triangles, focusing on special right triangles and the trigonometric ratios of sine, cosine, and tangent. Students will learn the definitions of the sine, cosine, and tangent ratios and use these definitions to set up equations to find a missing side length when one acute angle and one side length is known. They will also learn to use inverse trigonometric functions to find the angle measures when only the side lengths are known. Students will extend their understanding of indirect measurement to include the employment of trigonometric ratios and angles of elevation and depression. Lastly, students will learn the Law of Sines and Law of Cosines to solve problems involving oblique triangles.

Sample Assignment: Indirect Measurement

Students will use four different indirect measurement techniques involving similar triangles to solve real-world problems. The first will involve using shadows to find the height of a tall object, such as a flagpole or building. The second will involve using similar triangles to find the distance across a pond or river. The third will involve using a square corner held up to eye level to measure the height of a tall object using right triangle similarity.

For the fourth method, students will make a simple clinometer with a protractor, string, paper clip and a straw. They will use this tool to measure the angle of elevation to the top of a tall object. They will also use a tape measure to measure their distance to the base of the object and the height of their eye. Then they will set up a trigonometric equation to find the height of the object.

From this assignment, students will learn to apply the concept of triangle similarity using the fact that similar triangles have proportional corresponding side lengths. They will take accurate measurements of known lengths, draw diagrams, show how the triangles in their diagrams are congruent, and set up proportions to find the unknown length. They will compare the four methods of indirect measurement and discuss the benefits and limitations of each method. They will discuss the accuracy of each method and possible measurement errors that can be made in the activity. They will also discuss how indirect measurement can be used in real-world situations.

Transformational Geometry

In this unit, students extend their understanding of transformational geometry learned in the middle grades. They first learn that there are four isometries--reflections, translations, rotations, and glide reflections-- that preserve the size and shape of a figure. This is used to establish congruency of two figures. That is, two figures are congruent if and only if there is an isometry that maps one figure to the other. Compositions of isometries are explored to discover that the composition of any two of these isometries is equivalent to one of the four basic isometries. Students will also explore the properties of similarity transformations, dilations, which either enlarge or reduce a figure according to a scale factor.

Sample Assignment:

Students will be given the coordinates of the vertices of a figure on the coordinate plane. They will then perform a given composition of transformations on the figure. They will then perform the same transformations on the original figure, but in reverse order. Students will answer the following questions:

- Compare the results of the two compositions. Does the order of transformations matter?
- Is the image congruent to the preimage? Is it similar? Explain how you know.

Students will then write directions for someone else to follow a composition of transformations and include the solution.

From this assignment, students will learn that the order of compositions does not matter; the resulting image will be the same. They will also learn that a composition of transformations will either result in a congruent figure (if congruence transformations are used) or a similar figure (if a dilation is used).

Perimeter, Area, and Volume

In this unit, students will extend their knowledge of basic area formulas to derive formulas for surface area of prisms, cylinders, pyramids, and cones. They will then use these formulas to solve problems. Students will then focus on the concept of volume of space figures and either derive or prove formulas, beginning with the volume of a prism and then extending this formula to the volume of cylinders, pyramids, and cones. Lastly, the formulas for surface area and volume of spheres will be explored and used to solve problems.

Sample Assignment:

Students will make models of a cube, a square pyramid, a cylinder and a cone from four provided net templates and compare the dimensions and models. Then they will fill their open-faced pyramids with rice and transfer its contents to the cube and fill their open--faced cones and transfer the contents to the cylinder. They will make conjectures about the relationship between the volume of a prism and a pyramid and the volume of a cone and a cylinder.
Students will learn that the volume of a pyramid is one third the volume of a prism with the same size base and same height. They will also learn that the volume of a cone is one third the volume of a cylinder with the same size base and height.

Circles

In this unit, students will explore concepts related to circles. Students will find the length of part of a circle's circumference by relating it to an angle in the circle. They will also find the area of parts of a circle formed by radii and arcs when the circle's radius is known. Students will use their understanding of congruent triangles to prove statements about tangent lines. Students can determine characteristics of circumscribed figures using characteristics of tangent lines. Students will then broaden their understanding of special segments in circles to include congruent chords and congruent arcs. They will learn that congruent chords are equidistant from the center of a circle and that a diameter that is perpendicular to a chord bisects the chord and its related arc. Students learn the relationship between a central angle and its intercepted arc as well as the relationship between an inscribed angle and its intercepted arc. Corollaries from this theorem lead to observations about congruent inscribed angles, right angles within circles, and the angles of an inscribed quadrilateral. Lastly, students will learn that there are special relationships between intersecting chords, intersecting secants, or a secant and tangent that intersect.

Sample Assignment:

Students will work in groups to prepare and present an explanation of one of the theorems in this unit. This presentation should include a proof of the theorem, a drawn example, and a demonstration of how to solve a problem involving the theorem.

Examples of theorems to consider:
- The measure of an inscribed angle is half the measure of its intercepted arc.
- Two inscribed angles that intercept the same arc are congruent.
- An angle inscribed in a semicircle is a right angle.
- The opposite angles of a quadrilateral inscribed in a circle are supplementary.
- The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc.

Students will thoroughly learn the theorem that is assigned to their group. They will also practice constructing viable arguments and critiquing the reasoning of others by developing a proof of a theorem together with their group. Students will learn other theorems as they listen to the group presentations, take notes, and solve problems.

Probability

Students expand on their knowledge of probability to measure the likelihood that an event will occur. They will learn counting techniques to find all of the possible ways to complete different tasks or choose items from a list. Probability of compound events can be found by using the probability of each part of a compound event. Tables can be used to organize data by frequency and find probabilities. Tables, tree diagrams, and formulas are also used to find conditional probabilities. After studying basic probability concepts, students analyze how the concepts apply in real-world contexts.

Sample Assignment:

Working in small groups, each group decides on two different categories that their classmates would fall into based on a single characteristic. Sample categories include plays/does not play sports, has/does not have a part-time job, plays/does not play a musical instrument. Groups use Males and Females as the other category headings, then gather data and complete a two-way frequency table and a probability distribution for their data. They will then use the tables to calculate simple probabilities and compound probabilities.

By gathering their own data and constructing their own two-way frequency table, students will learn and understand the meaning of each entry in a two-way frequency table. They will also learn to use the table to calculate simple and compound probabilities.
Course Materials

Textbooks

Title	Author	Publisher	Edition	Primary
Holt McDougal Geometry , Common Core Edition©2012	Edward B. Burger, David J. Chard, Paul A. Kennedy, Steven J. Leinwald, Freddie L. Renfro, Tom W. Roby, Dale G. Seymour, Bert K. Waits	Houghton Mifflin Harcourt	2012	Yes
HMH Geometry: Student Edition 2015	Kanold, Burger, Dixon, Larson, Leinwand	Houghton Mifflin Harcourt	2015	Yes
Eureka Math: Geometry	Alkire, Allwood, et.al	Great Minds	2019	Yes
Pearson: Geometry, Common Core Edition ©2015	Charles, Kennedy, Hall	Pearson Education, Inc. / Prentice Hall	2015	Yes
Glencoe Geometry © 2018	Carter; Cuevas; Day; Malloy	McGraw Hill / Glencoe Publishing	2018	Yes

Websites

Title	Affiliated Institution or Organization	URL
Holt McDougal Online	Houghton Mifflin Harcourt	my.hrw.com